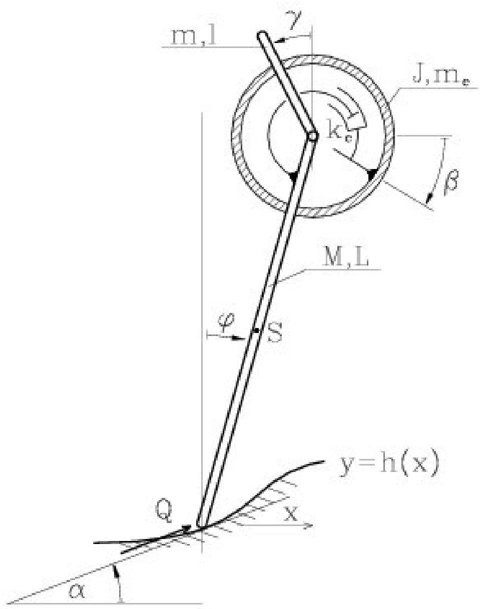
Dynamics of controlled mechanical systems considering sampling and processing delays

Stabilization of unstable equilibria of mechanical systems is frequently needed in engineering practice. A number of applications can be found in this field, e.g. the articulated bus powered through the rear axle, the shimmying wheels of trailers and airplanes, the thrust control of aircrafts, the control of cranes during weightlifting operations, the steering problem of motorcycles or the standing and walking robots. In the digital control of such systems, time delay always appears due to the sampling of the computer and to the time needed to process the measured signal. If the delay exceeds a certain critical value then the desired equilibrium of the controlled system becomes unstable. A typical basic example of controlling unstable equilibria by control force is the balancing. The control strategies of existing balancing models need an absolute reference line which serves as vertical direction. A balancing model has been constructed where the control strategy uses relative angular velocities instead of absolute angles and angular velocities. As a result, balancing works successfully also on a surface of unknown inclination.



Many mechanical problems lead to strongly nonlinear systems due to the piecewise linear terms in their governing equations of motion. Gear pairs with backlash, impact dampers, moving parts with dry friction and adjacent structures during earthquake are modelled by systems with piecewise linear stiffness, damping or restoring force. These phenomena lead to strongly nonlinear governing equations, so analytical solution techniques which are applicable to weakly nonlinear equations are not suitable for their analysis. A model of the inverted pendulum on a cart has been developed considering backlash at the driving teeth belt and the sampling and processing delays in the control system. A simplified one dimensional mathematical model has also been constructed which has analogous mathematical properties as the examined higher dimensional model. The orbits of this map represent a motion which can occur in piecewise linear systems with sampling.

Main achievements

“Artificial labyrinth”, an improved model of balancing based on the human balancing organ

- An improved balancing model, the “artificial labyrinth”, is constructed according to the structure and functioning of the human balancing organ. Using this model, digital balancing is successful without knowing a reference vertical direction. The stability conditions of the upper equilibrium of this model and the critical sampling delay when balancing becomes impossible are determined.

- The stability conditions of the upper equilibrium of the digitally controlled inverted pendulum on a cart and the critical sampling delay are determined. Backlash and sampling delay together result in stable but not periodic motions with finite amplitude.
- A one dimensional map is considered which has analogous mathematical properties as digitally controlled piecewise linear systems. The arising motion is more complicated than quasiperiodic, but it does not satisfy all the conditions of chaos. Due to its properties this motion may be called "marginally chaotic".

Selected publications on the topic

- Kollar, L. E., Stepan, G., Turi, J., Dynamics of Piecewise Linear Discontinuous Maps, *Int. J. of Bifurcation and Chaos*, Vol. 14, No. 7, pp. 2341-2351, 2004. IF (2004): 1.019
- Kollar, L. E., Stepan, G., Turi, J., Dynamics of Delayed Piecewise Linear Systems, *Electronic Journal of Differential Equations*, Conference 10, pp. 163-185, 2003. (presented at the *Fifth Mississippi State Conference on Differential Equations and Computational Simulations*, Starkville, MS, USA) (http://ejde.math.swt.edu or http://ejde.math.unt.edu)
- Stepan, G., Kollar, L. E., Balancing with Reflex Delay, *Mathematical and Computer Modelling*, Vol. 31, pp. 199-205, 2000. IF (2000): 0.387